電路學 (一)　第四次測驗　　四電機二 A　　2018 年 1 月 11 日　(星期四)

1) 滿分 100 分，考試時間 2 小時。
2) 答案應有正確之數值與單位。
3) 可使用計算器，但不得使用電腦、行動電話等通訊器材。不得參閱任何書本及筆記。
4) 請確實遵守考試規則，違反考試規則者，依本校校規處置。

1. The switch in the circuit shown in **Fig. 1** moves from position 1 to position 2 at t=0. Find $v_o(t)$ for $t > 0$. (20%)

2. In the critically damped circuit of **Fig. 2**, the initial conditions are $i_L(0)$= 2 A and $v_C(0)$= 5 V. Find the voltage $v(t)$, for $t > 0$. (20%)

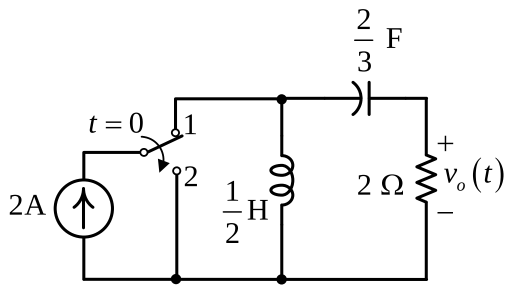


Fig. 1

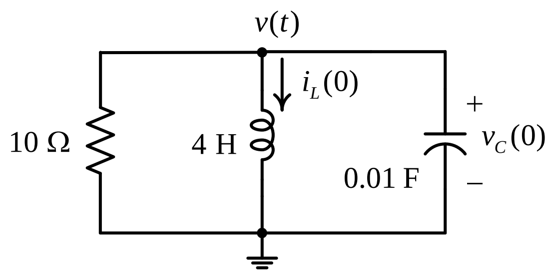


Fig. 2

3. The circuit of **Fig. 3** has been in steady-state before the switch moves from position 1 to position 2 at t=0. Find the voltage $v(t)$ for $t > 0$. (20%)

4. The switch in the circuit of **Fig. 4** closes at t = 0. If all the initial conditions are zero, find $v_C(t)$ for $t > 0$. (20%)

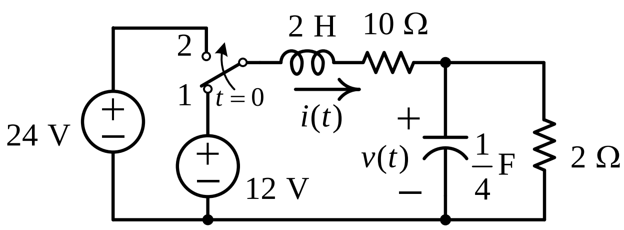


Fig. 3

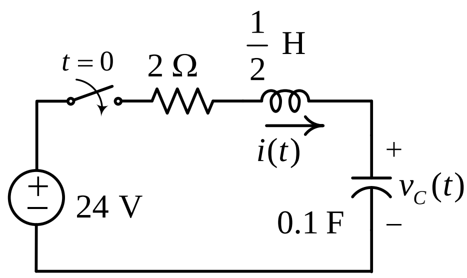


Fig. 4

5. Three branch currents in a network are known to be

$i_1(t) = 2\,sin(377t + 45°)$ A ,

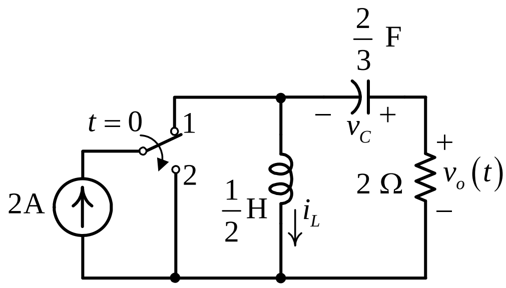
$i_2(t) = 0.5\,cos(377t + 10°)$ A ,

$i_3(t) = -0.25\,sin(377t + 60°)$ A .

Determine the phase angles by which $i_1(t)$ leads $i_2(t)$, and $i_1(t)$ leads $i_3(t)$. (20%)

Possible solutions to the final exam, 11th January 2018

1. After the switch moves from position 1 to 2, the problem becomes solving a series *R*-*L*-*C* circuit without source, given the initial conditions $i_L(0)$=2 A and $v_C(0)$=0. The KVL equation can be written as



$$\frac{1}{2}\frac{di_L}{dt}+2i_L+v_C=0\text{, with}$$

$$i_L=\frac{2}{3}\frac{dv_C}{dt}.$$

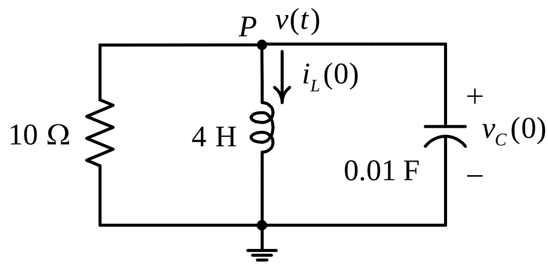
Combining the above two equations yields a 2nd-order ODE*

$\frac{1}{3}\frac{d^2v_C}{dt^2}+\frac{4}{3}\frac{dv_C}{dt}+v_C=0$, which allows the characteristic equation to be obtained as $s^2+4s+3=(s+1)(s+3)=0$. The natural frequencies are then s=–1 and s=–3 s^{-1}.

The current i_L can be assumed to be $i_L=Ae^{-t}+Be^{-3t}$. With the initial condition, we obtain the 1st equation $i_L(0)$=2 =A+B. The 2nd equation is given by

$\left.\frac{di_L}{dt}\right|_{t=0}=-A-3B=-4i_L(0)-2v_C(0)=-8$. Solving the two equations of A and B gives A=–1 and B=3. $\Rightarrow i_L=-e^{-t}+3e^{-3t}$ A $\Rightarrow v_o=-2i_L=2e^{-t}-6e^{-3t}$ V, $t>0$.

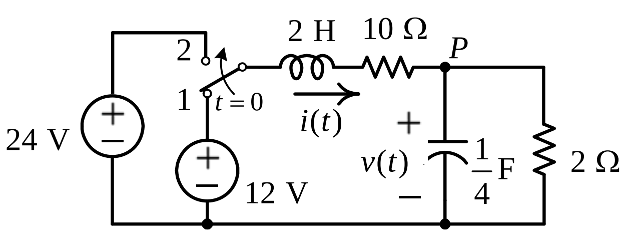
2. Writing the KCL equation at node P yields $\frac{v}{10}+i_L+0.01\frac{dv}{dt}=0$. Substituting $v=4\frac{di_L}{dt}$ to the KCL equation gives a 2nd-order ODE $\frac{d^2i_L}{dt^2}+10\frac{di_L}{dt}+25=0$, which gives the characteristic equation $s^2+10s+25=(s+5)^2=0$. A repeated root of s=–5 s^{-1} is found. The circuit is hence critically damped, and $v(t)$ must have the form $v=Ae^{-5t}+Bte^{-5t}$.



$$v(0)=A=5,\quad 0.01\left.\frac{dv}{dt}\right|_{t=0}=0.01(-5A+B)=-\left(i_L(0)+\frac{v(0)}{10}\right)=-2.5\Rightarrow B=-225$$

$$v=5e^{-5t}-225te^{-5t}\text{ V, }t>0.$$

3. Initial conditions $i(0)=\frac{12}{10+2}=1$ A , $v(0)=12\times\frac{2}{10+2}=2$ V. After the switch moves from position 1 to 2, the KVL equation is given by $-24+2\frac{di}{dt}+10i+v=0$,



and the KCL equation at node P is $i=\frac{1}{4}\frac{dv}{dt}+\frac{v}{2}$. Substitution of i in KCL for i in KVL gives a 2nd-order ODE of v

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* ODE: ordinary differential equation 常微分方程式

$\frac{d^2v}{dt^2} + 7\frac{dv}{dt} + 12v = 48$ with the characteristic equation $s^2 + 7s + 12 = (s+3)(s+4) = 0$. The voltage $v(t)$ must have the form $v = Ae^{-3t} + Be^{-4t} + k_p$.

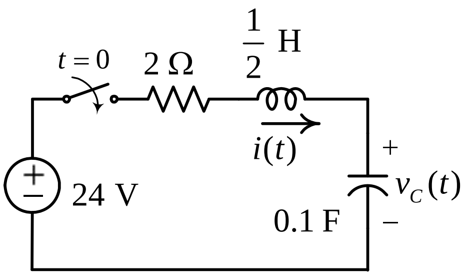
The final value $v(\infty) = 24 \times \frac{2}{10+2} = 4 \text{ V} = k_p$.

The initial value $v(0) = 2 = A + B + 4 \Rightarrow A + B = -2$

The initial value $i(0) = 1 = \frac{1}{4}\frac{dv}{dt}\Big|_{t=0} + \frac{v(0)}{2} = \frac{1}{4}(-3A - 4B) + \frac{2}{2} \Rightarrow 3A + 4B = 0$

Solving for A and B yields A=–8, B=6. $v = -8e^{-3t} + 6e^{-4t} + 4 \text{ V}$, t>0

4. The KVL equation is given by $-24 + 2i + \frac{1}{2}\frac{di}{dt} + v_C = 0$.



The current i can be expressed as $i = 0.1\frac{dv_C}{dt}$, which allows the KVL equation to be written as a 2nd-order ODE $\frac{d^2v_C}{dt^2} + 4\frac{dv_C}{dt} + 20v_C = 480$.

The characteristic equation is $s^2 + 4s + 20 = 0 \Rightarrow s = -2 \pm j4 \text{ s}^{-1}$. The capacitor voltage v_C must have the form $v_C = e^{-2t}(A\cos 4t + B\sin 4t) + k_p$.

The final value $v_C(\infty) = 24 = k_p$.

The initial value $v_C(0) = 0 = A + 24 \Rightarrow A = -24$.

The initial value $i(0) = 0 = 0.1\frac{dv_C}{dt}\Big|_{t=0} \Rightarrow -2A + 4B = 0 \Rightarrow B = -12$

$v_C = -e^{-2t}(24\cos 4t + 12\sin 4t) + 24 \text{ V}$, t>0

5. Transform all the currents in cosine functions.

$i_1(t) = 2\sin(377t + 45°) = 2\cos(377t + 45° - 90°) = 2\cos(377t - 45°)$ A

$i_2(t) = 0.5\cos(377t + 10°)$ A

$i_3(t) = -0.25\sin(377t + 60°) = -0.25\cos(377t + 60° - 90°)$

$= 0.25\cos(377t + 60° - 90° + 180°) = 0.25\cos(377t + 150°)$ A

$i_1(t)$ leads $i_2(t)$ by –45º–10º =–55º,

$i_1(t)$ leads $i_3(t)$ by –45º–150º =–195º or by –195º+360º=165º